

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 (a) $z = \sqrt{x + y^2}$

(i) Calculate the value of z when $x = 136$ and $y = \frac{9}{2}$.

$$z = \sqrt{x + y^2}$$
$$z = \sqrt{136 + \left(\frac{9}{2}\right)^2}$$
$$z = 12.5$$

Answer**12.5** [1]

(ii) Express x in terms of z and y .

$$z = \sqrt{x + y^2}$$
$$z^2 = x + y^2 \text{ **M1**}$$
$$x = z^2 - y^2 \text{ **A1**}$$

Answer $x = z^2 - y^2$ [2]

(b) Factorise $x^2 - 5x - 6$.

$$x^2 - 5x - 6 = (x - 6)(x + 1)$$

Answer $(x - 6)(x + 1)$ [2]

- 2 (a) Figure 1 shows a solid cone with radius 5 cm and slant height 13 cm.

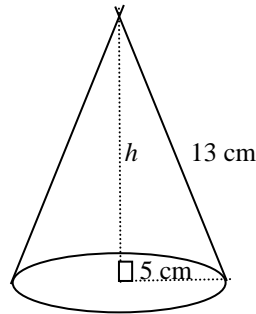


Figure 1

Calculate the

- (i) height, h , of the cone,

Use Pythagoras' theorem,

$$13^2 = h^2 + 5^2$$

$$169 - 25 = h^2$$

$$h = 12\text{cm}$$

Answer**12**.....cm [1]

- (ii) volume of the cone.

$$\text{Vol} = \frac{1}{3} \times \pi \times 25 \times 12$$

$$= 314 \text{ cm}^3 \text{ (3 sf)}$$

Answer**314**.....cm³ [2]

- (b) The solid cone is melted down and poured into a cylindrical mould with base radius of 6 cm as shown in Figure 2.

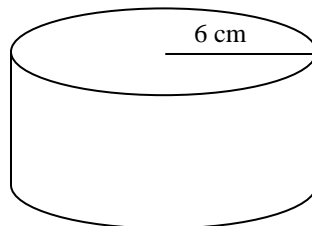


Figure 2

- (i) Calculate the height of the cylindrical mould.

Vol of cone = Vol of cylindrical mould

$$314.159 = \pi \times 36 \times h$$

$$h = 2\frac{7}{9} \text{ cm or } 2.78 \text{ cm (3s.f)}$$

Answer**2.78**.....cm [1]

- 2 (b) (ii) The solid is made from a material with density 1.2 g/cm^3 .

Calculate the mass of the solid.

$$\text{Since } \text{Density} = \frac{\text{Mass}}{\text{Volume}}, \text{Mass} = 1.2 \times 314.159$$

$$\approx 376.9908 \approx 377 \text{ g (3 s.f)}$$

Answer**377**.....g [2]

- 3 Solve the simultaneous equations.

$$2x + 3y = 9$$

$$4x - y = -17$$

$$2x + 3y = 9 \quad \text{----- (1)}$$

$$4x - y = -17 \quad \text{----- (2)}$$

$$\text{From (2)} \quad y = 4x + 17 \quad \text{----- (3)}$$

$$\text{Sub (3) into (1)} \quad 2x + 3(4x + 17) = 9$$

$$14x + 51 = 9$$

$$x = -3$$

$$\text{Sub } x = -3 \text{ into (3)} \quad y = 4(-3) + 17$$

$$y = 5$$

Answer $x = \dots\dots\dots -3 \dots\dots\dots$

$y = \dots\dots\dots 5 \dots\dots\dots$ [3]

- 4 (a) Factorise completely $x^2 - x - xy + y$.

$$x^2 - x - xy + y = x(x - 1) - y(x - 1)$$

$$= (x - y)(x - 1)$$

Answer $(x - y)(x - 1)$ [1]

- (b) Hence, express as a single fraction $\frac{2x}{x^2 - x - xy + y} - \frac{y}{x - 1}$.

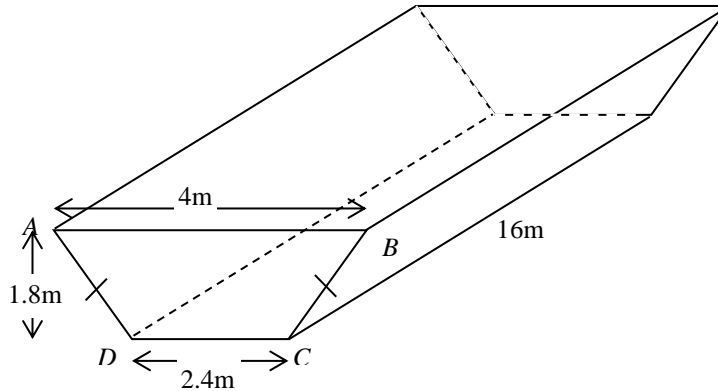
$$\frac{2x}{x^2 - x - xy + y} - \frac{y}{x - 1} = \frac{2x}{(x - y)(x - 1)} - \frac{y}{x - 1}$$

$$= \frac{2x - y(x - y)}{(x - y)(x - 1)}$$

$$= \frac{2x - xy + y^2}{(x - y)(x - 1)}$$

Answer $\frac{2x - xy + y^2}{(x - y)(x - 1)}$ [4]

- 5 A water tank, shown in the diagram, is in the shape of a prism. The trapezium, $ABCD$ is vertical.



Calculate

- (a) angle CBA ,

$$\tan \hat{CBA} = \frac{1.8}{0.8}$$

$$\hat{CBA} = \tan^{-1}\left(\frac{1.8}{0.8}\right)$$

$$\hat{CBA} = 66.0375^\circ$$

$$\hat{CBA} = 66.0^\circ$$

Answer 66.0° [2]

- (b) the area of trapezium $ABCD$,

$$\begin{aligned} \text{Area of trapezium } ABCD &= \frac{1}{2} \times (4 + 2.4) \times 1.8 \\ &= 5.76m^2 \end{aligned}$$

Answer $5.76m^2$ [2]

- (c) the volume of the water tank.

$$\begin{aligned} \text{Volume of water tank} &= 5.76 \times 16 \\ &= 92.16 \\ &= 92.2 m^3 \end{aligned}$$

Answer $92.2 m^3$ [2]

- 6 The table below shows the population, given to the nearest thousand, of some

countries.

Country	Population in 2019	Population in 2020
Korea	185 133 000	188 169 000
Japan	1 393 784 000	1 402 007 000
Australia	67 223 000	?

- (a) The population of Australia in 2020 was 6.74×10^7 .
Complete the table, expressing Australia's population in 2020 as an ordinary number.

$$6.74 \times 10^7 = 67\,400\,000$$

Answer67 400 000..... [1]

- (b) Calculate the difference in population between Japan and Korea in 2020.
Give your answer in standard form, correct to three significant figures.

$$\begin{aligned} \text{Difference} &= 1\,402\,007\,000 - 188\,169\,000 \\ &= 1\,213\,838\,000 \\ &= 1.21 \times 10^9 \end{aligned}$$

Answer1.21 x 10⁹..... [2]

7 y is inversely proportional to the square of x and $y = 3$ when $x = 8$.

- (a) Express y in terms of x .

$$\begin{aligned} yx^2 &= 3 \times 8^2 \\ y &= \frac{192}{x^2} \end{aligned}$$

Answer $y = \frac{192}{x^2}$ [2]

- (b) Find the new value of y when x increases by 100%.

$$\begin{aligned} y &= \frac{192}{16^2} \\ y &= \frac{3}{4} \end{aligned}$$

Answer $y = \frac{3}{4}$ [2]

8 \$450 is divided between Anna, Sally and Nina in the ratio of 5 : x : 4x.

(a) Express, as a single fraction in terms of x, the amount that Sally received.

$$\begin{aligned}\text{Amount Sally received} &= \frac{x}{5+x+4x} \times 450 \\ &= \frac{90x}{1+x}\end{aligned}$$

Answer $\frac{90x}{1+x}$ [2]

(b) Hence, given that Sally received \$85, evaluate Anna's share.

$$\begin{aligned}\frac{90x}{1+x} &= 85 \\ 90x &= 85 + 85x \\ x &= 17\end{aligned}$$

$$\begin{aligned}\text{Anna received} &= \frac{5}{90} \times 450 \\ &= \$25\end{aligned}$$

Answer \$25 [3]

9 Answer the whole of this question on a single sheet of graph paper.

The table below is for $y = 2x^2 - 5x - 3$.

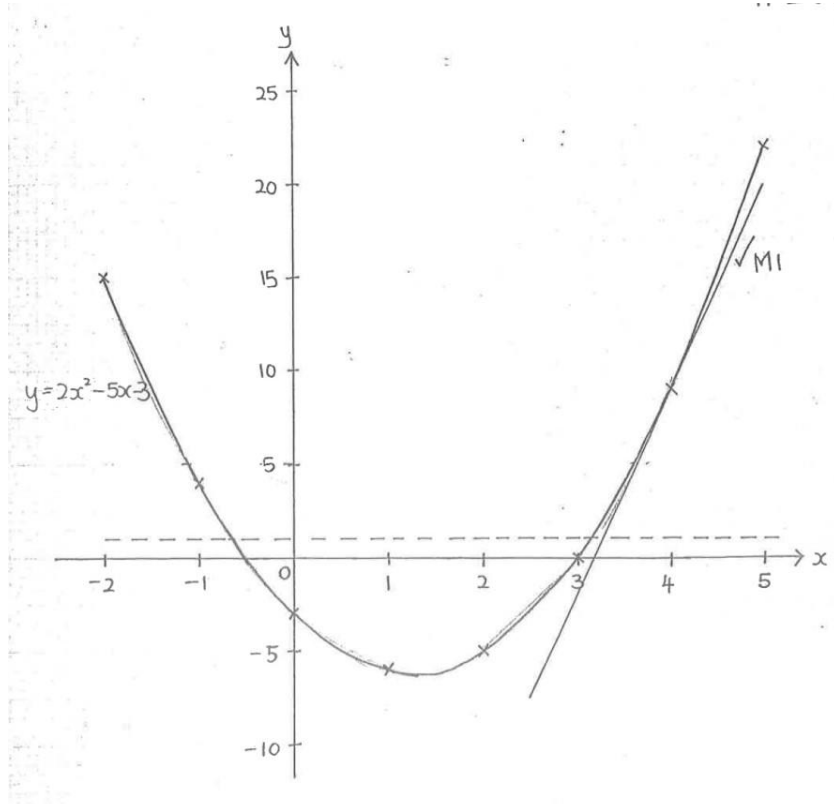
x	-2	-1	0	1	2	3	4	5
y	p	4	-3	-6	-5	0	9	22

(a) Find the value of p.

$$\begin{aligned}p &= 2(-2)^2 - 5(-2) - 3 \\ p &= 15\end{aligned}$$

Answer $p = \dots\dots 15 \dots\dots$ [1]

- (b) Using a scale of 2 cm to represent 1 unit on the x -axis and 2 cm to represent 5 units on the y -axis, draw the graph of y against x for the range $-2 \leq x \leq 5$.



[3]

- (c) From your graph, find the values of x when $y = 5$.

Answer $\dots\dots x = -1.1$ or $3.6 \dots\dots$ [2]

- (d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$.

$$\begin{aligned} \text{gradient} &= \frac{20 - (-7.5)}{5 - 2.5} \\ &= 11 \end{aligned}$$

Answer $\dots\dots 11 \dots\dots$ [2]

(e) From the graph, solve $2x^2 - 5x = 3$.

$$2x^2 - 5x = 3$$

$$2x^2 - 5x - 3 = 0$$

$$\therefore y = 0$$

From the graph, $x = -0.5 \text{ or } 3$

Answer $x = -0.5 \text{ or } 3$ [2]

10 The stem-and-leaf diagram shows the distribution of the ages of the contestants in a painting contest. The median and the range of the distribution are 32 and 22 years respectively.

Stem	Leaf
2	1 3 3
2	5 6 6 8 9
3	0 <i>m</i> 3 5 5 5
3	6 6 8 8
4	1 <i>n</i>

Key: 2|1 represents 21 years

Find

(i) the values of m and n ,

Answer $m = \dots\dots 1 \dots\dots\dots$

Answer $n = \dots\dots 3 \dots\dots\dots$ [2]

(ii) the modal age,

Answer 35 [1]

(iii) the interquartile range of the speed.

$$\text{Lower Quartile} = 26$$

$$\text{Upper Quartile} = 36$$

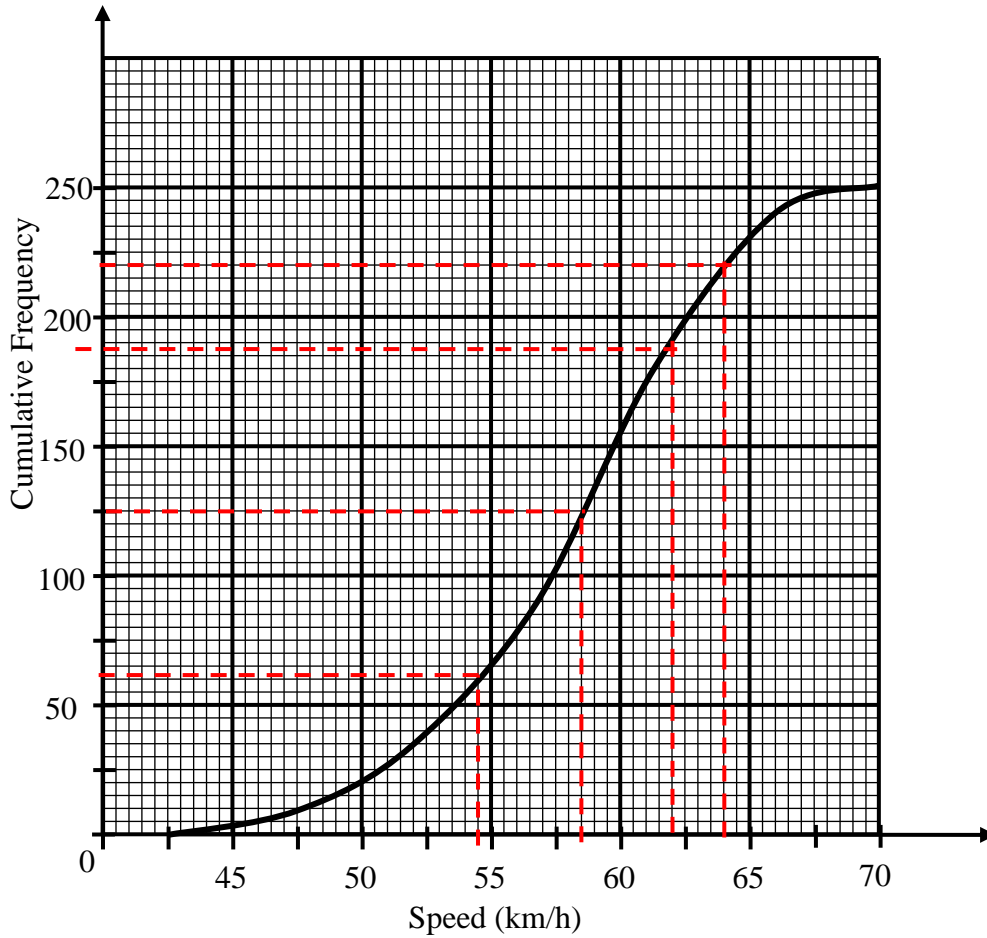
$$\text{Interquartile range} = 36 - 26 = 10$$

Answer 10 [1]

Section B (8 marks)

Answer **one** question from this section. Each question carries 8 marks.

- 10 (a)** The speeds, in km/h, of 250 vehicles travelling along a road were measured. The cumulative frequency curve summarises the results.



- (i)** Find the median speed.

$$\begin{aligned} \text{Median position} &= \frac{50}{100} \times 250 \\ &= 125 \end{aligned}$$

From the graph, median speed
= 58.5 km/h

Answer = 58.5 km/h [1]

- (ii) Find the interquartile range of the speed.

$$\begin{aligned}\text{Upper quartile position} &= \frac{3}{4} \times 250 \\ &= 187.5\end{aligned}$$

$$\begin{aligned}\text{From the graph, Upper quartile speed} \\ &= 62 \text{ km/h}\end{aligned}$$

$$\begin{aligned}\text{Lower quartile position} &= \frac{1}{4} \times 250 \\ &= 62.5\end{aligned}$$

$$\begin{aligned}\text{From the graph, Lower quartile speed} \\ &= 54.5 \text{ km/h}\end{aligned}$$

$$\begin{aligned}\text{Interquartile Range} \\ &= 62 - 54.5 \\ &= 7.5\end{aligned}$$

Answer7.5..... [2]

- (iii) Find the speed limit, in km/h, if 12% of the vehicles were found speeding.

$$\begin{aligned}\frac{12}{100} \times 250 &= 30 \text{ cars found speeding} \\ 220 \text{ cars below speed limit} \\ \text{From graph, speed limit} \\ &= 64 \text{ km/h}\end{aligned}$$

Answer64.....km/h [2]

- (b) A bag contains 5 red balls, 6 blue balls and 1 green ball.
Two balls are drawn from the bag, one after the other, without replacement.

Calculate the probability that the two balls have

- (i) the same colour,

$$\begin{aligned}\text{Probability(same colours)} &= \frac{5}{12} \times \frac{4}{11} + \frac{6}{12} \times \frac{5}{11} + \frac{1}{12} \times \frac{0}{11} \\ &= \frac{25}{66}\end{aligned}$$

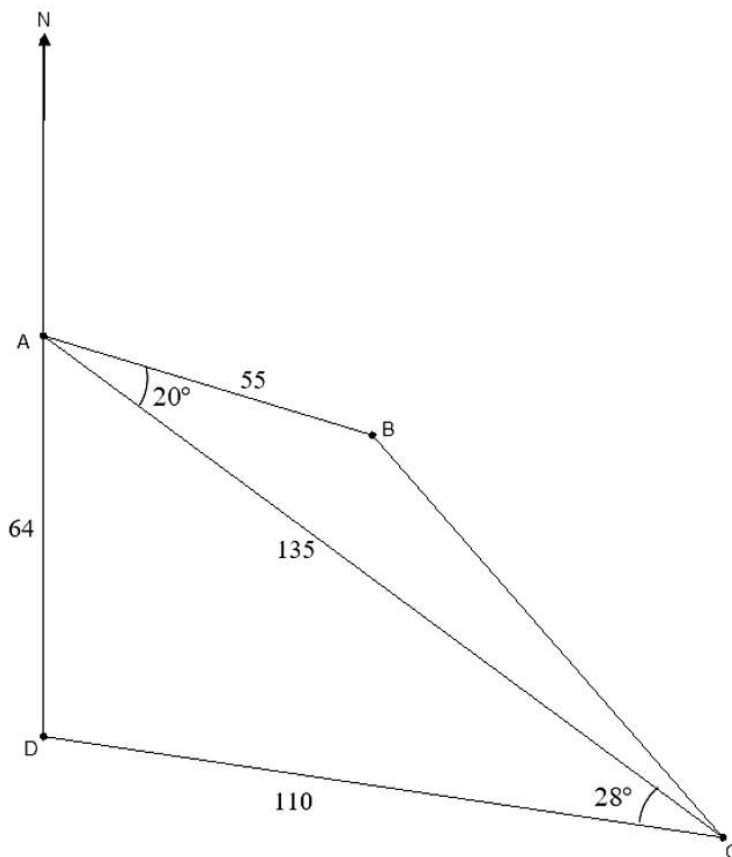
Answer $\frac{25}{66}$ [2]

(ii) different colours.

$$\begin{aligned}
 &P(\text{different colours}) \\
 &= 1 - \frac{25}{66} = \frac{41}{66}
 \end{aligned}$$

Answer $\frac{41}{66}$ [1]

11 A, B, C and D are four points on a grass field. $AB = 55$ m, $AC = 135$ m, $CD = 110$ m, and $AD = 64$ m. Angle $ACD = 28^\circ$, angle $CAB = 20^\circ$ and D is due south of A .



(a) Find the distance BC .
By cosine rule,

$$BC = \sqrt{55^2 + 135^2 - 2(55)(135)(\cos 20^\circ)}$$
$$= 85.4m(3s.f)$$

Answer**85.4**.....m [2]

- (b) A building is erected at point C . The angle of elevation from point D is 20° .
What is the height of the building?

$$\tan 20^\circ = \frac{\text{height}}{110}$$
$$\text{height} = 40.0m$$

Answer40.0m [2]

- (c) Find the bearing of C from A .

By sine rule,

$$\frac{\sin \angle CAD}{110} = \frac{\sin 28^\circ}{64}$$
$$\angle CAD = 53.79^\circ(2dp)$$

$$180^\circ - 53.79^\circ$$
$$\text{Bearing of } C \text{ from } A = 126.21^\circ$$
$$= 126.2^\circ(1dp)$$

Answer**126.2**..... $^\circ$ [3]

- (d) Find the area enclosed by points A , B and C .

$$\text{Area of triangle } ACD = \frac{1}{2} \times 110 \times 135 \times \sin 28^\circ$$
$$= 3490m^2$$

Answer3490..... m^2 [1]

End of paper