

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 (a) Evaluate the following to 1 significant figure

$$\sqrt{\frac{35.45 \times 105.96}{9.56}}$$

$$\sqrt{\frac{40 \times 100}{10}}$$

$$= \sqrt{\frac{4000}{10}}$$

$$= \sqrt{400}$$

$$= 20$$

Answer**20**..... [2]

- (b) Find the percentage difference between the value found in part (a) and the actual value of $\sqrt{\frac{35.45 \times 105.96}{9.56}}$, correcting your answer to 2 decimal places.

$$\sqrt{\frac{35.45 \times 105.96}{9.56}} = 19.82212$$

$$\% \text{ difference} = \frac{20 - 19.82212}{19.82212}$$

$$= 0.0947302\%$$

$$= 0.09\% \text{ (2d.p)}$$

Answer**0.09%** [2]

- 2 A regular n-sided polygon has interior angle of 135° . Find n.

$$\frac{(n-2) \times 180}{n} = 135$$

$$180n - 360 = 135n$$

$$180n - 135n = 360$$

$$45n = 360$$

$$n = 8$$

Alternate

$$\text{exterior} \square = 180 - 135$$

$$= 45$$

$$n = 360 \div 45$$

$$n = 8$$

Answer**8**..... [2]

3 Furniture cost \$6680.

Mr Koh paid a 25% deposit and then \$220 per month for 2 years.

How much did he pay in total?

Working:

$$\begin{aligned}\text{Deposit} &= 6680 \times \frac{25}{100} \\ &= \$1670 \quad \text{(M1)}\end{aligned}$$

$$\begin{aligned}\text{Total payment} &= 1670 + (220 \times 24) \\ &= \$6950 \quad \text{(A1)}\end{aligned}$$

Answer\$6950..... [2]

4 (a) Find the highest common factor of 40 and 300.

Working:

$$\begin{aligned}40 &= 2^3 \times 5 \\ 300 &= 2^2 \times 3 \times 5^2 \quad \text{(M1)}\end{aligned}$$

$$\begin{aligned}\text{HCF} &= 2^2 \times 5 \\ &= 20 \quad \text{(A1)}\end{aligned}$$

Answer20..... [2]

(b) The square root of k is $2 \times 3^2 \times 5$

Find k as the product of its prime factors.

Answer..... $2^2 \times 3^4 \times 5^2$ [1]

5 x is an even number.

The sum of the next two consecutive numbers has a maximum value of 135.

(a) Form an inequality in terms of x .

$$\text{Answer} \dots\dots (\mathbf{x+1})+(\mathbf{x+2})\leq\mathbf{135}.. [1]$$

(b) Find the largest possible even number for x .

Working:

$$(x+1)+(x+2)\leq 135$$

$$2x+3\leq 135 \quad \text{(M1)}$$

$$2x\leq 132$$

$$x\leq 66 \quad \text{(A1)}$$

$$\text{Answer} \dots\dots \underline{\mathbf{x=66}}\dots\dots\dots [2]$$

6 (a) Simplify $y \div \frac{y^2}{5}$

$$\text{Answer} \dots\dots\dots \frac{\mathbf{5}}{\mathbf{y}}\dots\dots\dots [1]$$

(b) Expand and simplify $5y\left(x + \frac{3y}{5}\right) + 2y(3-x)$

Working:

$$5y\left(x + \frac{3y}{5}\right) + 2y(3-x) = 5xy + 3y^2 + 6y - 2xy \quad \text{(M1)}$$

$$= 3xy + 3y^2 + 6y \quad \text{(A1)}$$

$$= \mathbf{3y^2 + 6y + 3xy}$$

(Accept $3y(y+2+x)$ or equivalent)

$$\text{Answer} \dots\dots\dots \mathbf{3y^2 + 6y + 3xy} \dots\dots\dots [2]$$

7 Given that $16^x = 2^4 \times \sqrt{4^x}$, find x .

$$2^{4x} = 2^4 \times \sqrt{2^{2x}}$$

$$2^{4x} = 2^4 \times 2^{\frac{2x}{2}}$$

$$4x = 4 + x$$

$$x = \frac{4}{3}$$

Answer $x = \frac{4}{3}$ [3]

8 (a) Elaine saved \$50 000 in a bank. Elaine's savings was compounded at 3% for every 6 months.

Calculate the value of the savings after 10 years.

Value after 10 years

$$= 50000 \left(1 + \frac{1.5}{100} \right)^{20}$$

$$= \$67342.75$$

Answer \$67342.75 [3]

(b) John borrowed a sum of money from a bank at 9.8% per year simple interest for 2 years. He repaid the amount of \$2499 after interest.

Find the sum of money borrowed.

$$2499 = \frac{P(9.8)(2)}{100}$$

$$P = \$12750$$

Answer \$12750 [2]

9 A formula is given as $T = \frac{4-x}{2x+1}$.

(a) Find T when $x = -3$.

$$T = \frac{4 - (-3)}{2(-3) + 1}$$

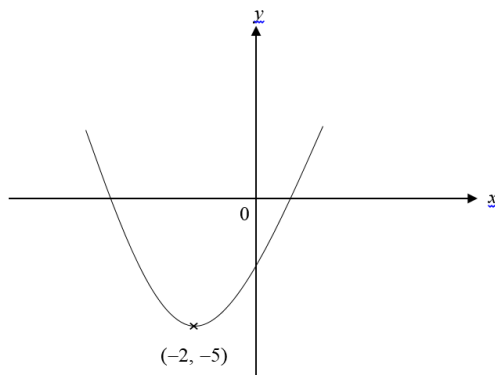
$$T = -\frac{7}{5}$$

Answer $T = -\frac{7}{5}$ [1]

(b) Express x in terms of T .

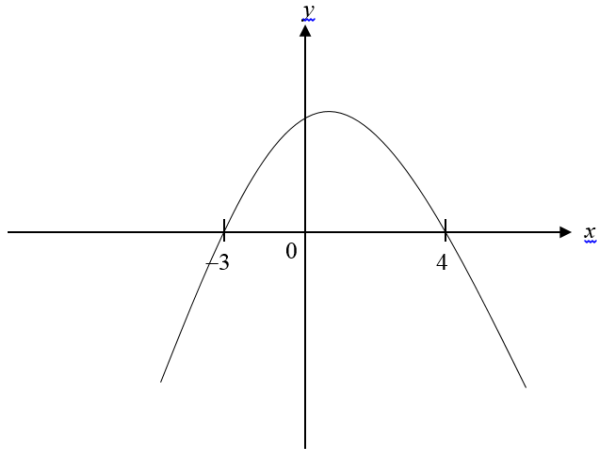
[2]

10 (a) Sketch the graph of $y = (x+2)^2 - 5$.



[2]

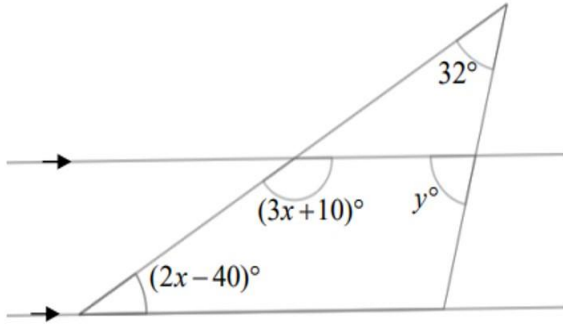
(b) (i) Sketch the graph of $y = (x + 3)(4 - x)$.



(ii) State the equation of the line of symmetry of $y = (x + 3)(4 - x)$. [2]

Answer $x = 0.5$ [1]

11



(a) Calculate the value of x .

$$2x - 40 + 3x + 10 = 180$$

$$5x = 210$$

$$x = 42$$

Answer $x = 42$ [2]

(b) Calculate the value of y .

$$y = 32 + (2(42) - 40)$$

$$y = 76$$

Answer $y = 76$ [2]

12 The distance between two towns on a map is 31.2 cm.

The scale of the map is 1 : 20 000.

(a) Find the actual distance between the towns. Give your answer in kilometres.

$$1 \text{ cm} : 0.2\text{km} \quad [\text{M1} - \text{convert to km}]$$

$$31.2 \div 0.2 = 6.24$$

Answer **6.24** km [2]

- (b) The actual area of a field is 300 000 m².
Find the area of the field on the map in cm².

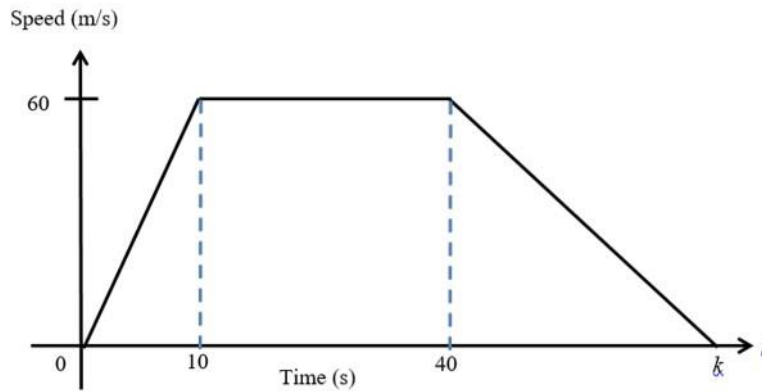
$$1 \text{ cm} : 200\text{m}$$

$$1\text{cm}^2: 40\,000 \text{ m}^2$$

$$\frac{300000}{40000} = 7.5\text{cm}^2$$

Answer7.5..... cm² [2]

- 13 The diagram below shows the speed-time graph of a car over a period of time.



- (a) Describe the movement of the car between $t = 10$ and $t = 40$.

Answer:

..... The car is moving at constant speed of 60 m/s.

..... [1]

- (b) Calculate the acceleration of the car for the first 10 seconds.

$$acc = \frac{60}{10} = 6\text{ m/s}^2$$

Answer6..... m/s² [1]

- (c) Evaluate the distance travelled in the first 40 seconds.

$$\begin{aligned} \text{area under graph} &= \frac{1}{2} \times (40 + 30) \times 60 \\ &= 2100\text{m} \end{aligned}$$

Answer**2100**..... m [2]

- (d) Calculate the value of k for the car to come to rest if the deceleration is 3m/s^2 .

$$\begin{aligned} -3 &= \frac{60}{40 - k} \\ -3(40 - k) &= 60 \\ -120 + 3k &= 60 \\ 3k &= 180 \\ k &= 60 \end{aligned}$$

Answer $k = \dots$ **60**..... [2]

- 14** Brandon has a bag containing 4 blue balls and 3 red balls.
He selects a ball at random.
He returns the first ball into the bag and selects a second ball at random.

What is the probability that Brandon chooses

- (a) two balls of the same colour,

$$\frac{4}{7} \times \frac{4}{7} + \frac{3}{7} \times \frac{3}{7} = \frac{25}{49}$$

Answer $\frac{25}{49}$ [2]

- (b) one ball of each colour?

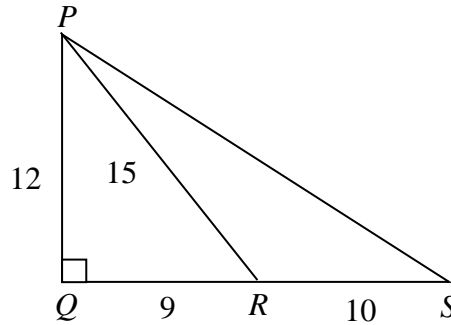
$$1 - \frac{25}{49} = \frac{24}{49}$$

Alternative:

$$\frac{4}{7} \times \frac{3}{7} + \frac{3}{7} \times \frac{4}{7} = \frac{24}{49}$$

Answer $\frac{24}{49}$ [2]

- 15** In the diagram below, QRS is a straight line and $\angle PQR = 90^\circ$.
 $PQ = 12$ cm, $QR = 9$ cm, $PR = 15$ cm and $RS = 10$ cm.



- (a) Find the length of PS .

$$PS = \sqrt{12^2 + 19^2}$$

$$= 22.5 \text{ cm}$$

Answer22.5.....cm [1]

- (b) Write as a fraction

- (i) $\tan \angle QSP$,

$$\tan \angle QSP = \frac{12}{19}$$

Answer $\frac{12}{19}$ [1]

- (ii) $\sin \angle QPR$,

$$\sin \angle QPR = \frac{9}{15} = \frac{3}{5}$$

Answer $\frac{3}{5}$ [1]

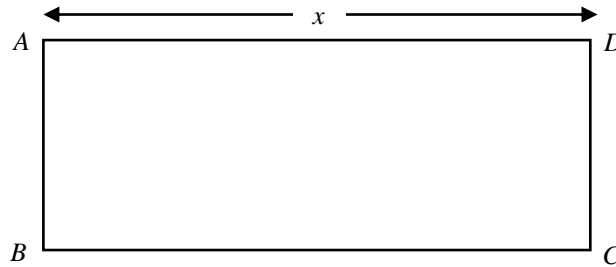
- (iii) $\cos \angle PRS$.

$$\cos \angle PRQ = \frac{9}{15} = \frac{3}{5}$$

$$\cos \angle PRS = -\frac{3}{5}$$

Answer $-\frac{3}{5}$ [1]

- 16** The diagram shows a rectangle, $ABCD$, with $AD = x$ centimetres.
The perimeter of the rectangle is 100 cm.



- (a)** Write an expression in its simplest form, in terms of x , for

- (i)** the length of AB ,

$$50 - x$$

Answer $50 - x$ *cm* [1]

- (ii)** the area of the rectangle,

$$50x - x^2$$

Answer $50x - x^2$ *cm²* [1]

- (b)** The area of the rectangle is 350 cm^2 .
Show that $x^2 - 50x + 350 = 0$.

Answer

$$50x - x^2 = 350$$

$$50x - x^2 - 350 = 0$$

$$x^2 - 50x + 350 = 0$$

Answer $x^2 - 50x + 350 = 0$ [2]

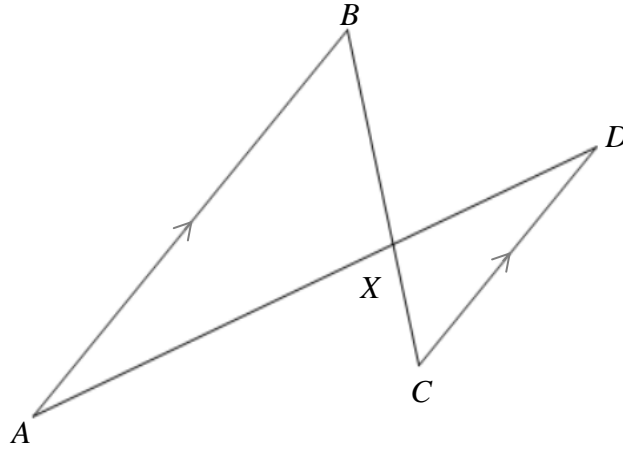
- (c)** Solve the equation $x^2 - 50x + 350 = 0$ to find the value of x if $AD > AB$.

$$x = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(350)}}{2(1)}$$

$$x = 41.6 \text{ or } x = 8.42 \text{ (reject)}$$

Answer $x = \dots x = 41.6 \text{ or } x = 8.42 \text{ (reject)}$ [3]

- 17** Triangle ABX is similar to triangle DCX .
 $AB = 12.5$ cm, $CD = 7.5$ cm and $AX = 9$ cm.



- (a) Calculate the length of XD .

$$\frac{XD}{XA} = \frac{CD}{BA}$$

$$\frac{XD}{9} = \frac{7.5}{12.5}$$

$$XD = 5.4\text{cm}$$

Answer5.4cm..... [2]

- (b) Find the scale factor for the enlargement of CXD to triangle BXA .

$$\frac{12.5}{7.5} = \frac{5}{3}$$

Answer $\frac{5}{3}$ [1]

- 18** The equation of the line l is $3x+4y=24$.

- (a) (i) State the gradient of the line.

$$3x + 4y = 24$$

$$4y = -3x + 24$$

$$y = -\frac{3}{4}x + 6$$

$$\text{Gradient} = -\frac{3}{4}$$

Answer $-\frac{3}{4}$ [1]

- (ii) Given that line l crosses the x -axis and y -axis at points A and B respectively, find the coordinates of the points A and B .

$$\begin{aligned}3x + 4y &= 24 \\x &= 0, \\4y &= 24 \\y &= 6 \\y &= 0 \\3x &= 24 \\x &= 8 \\A(8,0) \quad B(0,6)\end{aligned}$$

Answer $A (8 , 0)$
 $B (0 , 6)$ [2]

- (b) Find the area of the triangle AOB .

$$\begin{aligned}\text{Area of triangle } ABO \\&= \frac{1}{2} \times 8 \times 6 = 24 \text{ units}^2\end{aligned}$$

Answer**24**..... units² [2]

- (c) (i) Given that the gradient of another line h is 0.75 and it passes through the point $C(2, 5)$, find the equation of line h .

$$\begin{aligned}y &= \frac{3}{4}x + c \\5 &= \frac{3}{4}(2) + c \\c &= \frac{7}{2} \\y &= \frac{3}{4}x + \frac{7}{2}\end{aligned}$$

Answer $y = \frac{3}{4}x + \frac{7}{2}$ [2]

(ii) Find the length of BC .

$$\begin{aligned} \text{Length of } BC &= \sqrt{(5-6)^2 + (2-0)^2} \\ &= 2.24\text{units} \end{aligned}$$

Answer**2.24**..... units [2]

19 Observe the sequence below.

$$\begin{aligned} 1^2 - 0^2 &= 1 \\ 2^2 - 1^2 &= 3 \\ 3^2 - 2^2 &= 5 \\ \vdots \quad \vdots \quad \vdots & \end{aligned}$$

(a) Write down the 5th line of the sequence.

Answer**5² - 4² = 9**..... [1]

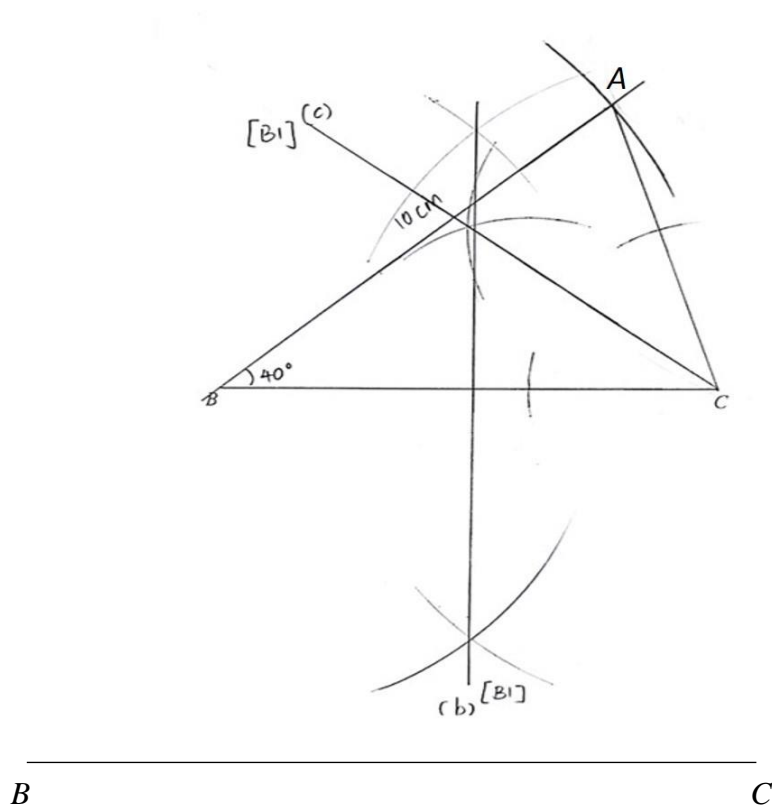
(b) Write down an expression, in terms of n , for the n^{th} line of the sequence.

Answer **$n^2 - (n-1)^2 = 2n - 1$** [1]

(c) Evaluate 25th line of the sequence.

Answer $25^2 - 24^2 = 49$ [1]

20 The line BC is drawn below.



(a) $AB = 10$ cm and $\hat{A}BC = 40^\circ$. Construct the triangle ABC . [1]

(b) Construct the perpendicular bisector of BC . [1]

(c) Construct the bisector of $\hat{A}CB$. [1]