2021 O-LEVEL MATHEMATICS – PAPER 2 (ANSWER KEY)

1a	i	$\frac{3}{4u^2-1}-\frac{1}{(1-2u)}$ $=\frac{3+(2u-1)}{(2u+1)(2u-1)}$ $=\frac{2u+2}{(2u+1)(2u-1)}$	M1 for common denominator A1 must be factorised denom
	ii	$\frac{(3x+2)(3x-2)}{(3x+2)(x-3)}\div(3x+2)$ $=\frac{(3x+2)(3x-2)}{(3x+2)(x-3)}\times\frac{1}{(3x+2)}$ $=\frac{3x-2}{(3x+2)(x-3)}$	M1 for difference of 2 squares M1 for factorised denominator A1 must be factorised denom
1b		$\frac{1}{g}=\frac{1}{u}+\frac{1}{v^2}$ $\frac{1}{g}-\frac{1}{u}=\frac{1}{v^2}$ $\frac{u-g}{gu}=\frac{1}{v^2}$ $v^2=\frac{gu}{u-g}$ $v=\pm\sqrt{\frac{gu}{u-g}}$	M1 for denominator *isolating* v^2 *A1 must have* $\pm$

2	a)	$\frac{PRT}{100} = 9683 - 9200$ $\frac{(9200)(1.05)T}{100} = 483$ $t = 5$	M1 for interest A1
	b)	$A = P\left(1 + \frac{r}{100}\right)^n$ $4030 = P\left(1 + \frac{\frac{3.2}{2}}{100}\right)^{20}$ P=$2934	 M1 for correct r & n A1
	c)	$\$1760 + 86.60 \times 6 \times 12$ $= \$7995.20$	M1 A1

3	a)	$\therefore \text{Fraction} = \frac{56}{160} = \frac{7}{20}$	A1
	b)	45% of 160 = 72 students or 55% of 160 = 88 students From the graph, x = 52	M1 A1
	c)	90% of 160 = 144 students 90th percentile = 70 marks	 A1
	d)	I do not agree. For the students who have the highest marks, they can be the <u>outliers (extreme values).</u> To compare the performance, median marks of each exam should be used instead. The median mark for the Science exams is higher than that of the Maths exams so the students performed better in the Science exams.	B1 A1

4	ai)	$\cos ADC = \frac{26.2^2+39^2-58^2}{2(26.2)(39)}$ $ADC = 124.468°$ $= 124.5°$	M1 A1
	aii)	128°	B1
	aiii)	$\angle CDB = 55.532°$ $\frac{BC}{sin\angle CDB} = \frac{26.2}{\sin(180°-52°-55.532°)}$ $BC = 22.65m$ $=22.7\ m$	M1 ecf A1
	aiv)	$\frac{1}{2}(39)(26.2)(sin124.468°)$ $= 421.2m^2$ $421\ m^2$	M1ecf A1

	b)	$\frac{1}{2} \times 58 \times h = 421.208m^2$ $h = 14.52\ m$ $= 14.5$ m	M1 A1
	c)	$tan\vartheta = \frac{6}{14.52}$ $\vartheta = 22.452°$	M1 A1

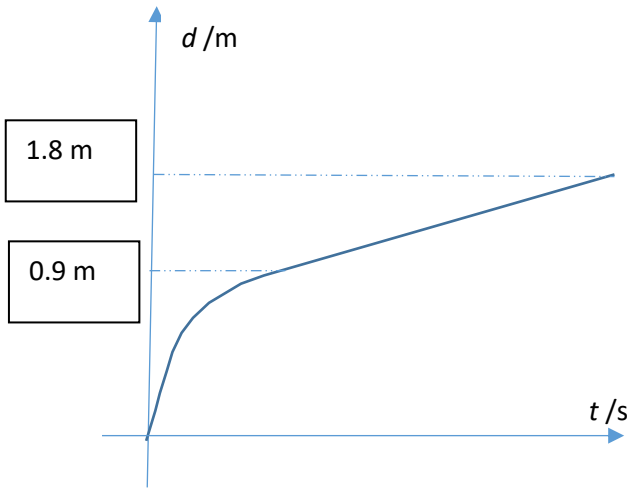
5	ai	$\angle CDO = \frac{\pi}{2}$(tangent perpendicular to rad)	B1
	aii	$\angle DCO = \pi - \frac{\pi}{2} - \frac{\pi}{6}$ (sum of angles in triangle) $=\frac{\pi}{3}$ $\angle BCD = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ $(angles\ in\ st\ line)$ Or $\angle BCD = \angle CDO + \angle DOC$ (ext angle=sum of int opp angles in triangle) $= \frac{\pi}{2} + \frac{\pi}{6}$ $= \frac{2\pi}{3} rad$ or 2.09	M1 for 1 reason A1 M1 for reason A1
5	bi	$\text{Sin}\frac{\pi}{6} = \frac{8}{OC}$ $OC = 16$ cm	M1 A1
	bii	Radius of OB=24 cm Length of arc $=24 \times \frac{\pi}{6}$ $= 12.566$ cm	M1 M1 A1
	biii	Arc BCD= $8\left(\frac{2\pi}{3}\right)$ $= 16.755$ $\text{Tan}\frac{\pi}{6} = \frac{8}{DO}$ $DO = 13.856$ $AD = 24 - 13.856$ $= 10.144$ $perimeter = 12.566 + 16.755 + 10.144$ $= 39.465$ $= 39.5\ cm$	M1 M1 for *DO* M1 A1
	biv	Area of the shaded region $=\frac{1}{2}(24^2)\left(\frac{\pi}{6}\right) - \frac{1}{2}(8^2)\left(\frac{2\pi}{3}\right) - \frac{1}{2}(8)(13.856)$ =28.4 cm^2	M1 A1

6a	p= 1.5625 or =1.56	B1
6b	Scale correct and axes labelled – B1 Smooth curve - C1 At least 5 correctly plotted points –P1	
6c	x = 1.55 to 1.75 or 4 to 4.3	B1 (both correct)
6d	$-10 + 3x + \frac{25}{x^2} < 0$ $y + 2 < 0$ $y < -2$ Curve is above -2, no solution.	B1 (no solution + must state some logical reasoning)
6e	Gradient (1.3, 2) & (2, 0.25) $= \frac{2 - 0.25}{1.3 - 2}$ =-2.5	Tangent line drawn B1 Gradient accept -5.5 to -1.5
6fi	Draw y=-x+3 correctly $\boldsymbol{x}$ = 1.55 to 1.85 or 2.95 to 3.2	B1 (check passes through, (1.5, 1.5) and (3,0) B1 (both correct)
6fii	$-12 + 3x + \frac{25}{x^2} = -x + 3$	B1 (or any equivalent)

7	**i**	$\frac{1400}{k} seconds$	**B1**
	ii	$\frac{1400}{k} + 3.5 \times 60 = 30 \times \frac{50}{k-0.2}$ $\frac{1400}{k} + 200 = \frac{1500}{k-0.2}$ $200 = \frac{1500x - 1400(k-0.2)}{k(k-0.2)}$ $200k^2 - 40k = 100k + 240$ $10k^2 - 7k - 14 = 0$ shown	$\frac{1400+200x}{k} = \frac{1500}{(k-0.2)}$ **M1 for converting min to seconds** **M1 for common denominator** **A1**
	iii	$k = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(10)(-14)}}{2(10)}$ $k = 1.58 \; or \; -0.88$	**M1** **A2 2 dp**
	iv)	$\frac{50}{1.584} = 31.6$ $\frac{50}{1.584 - 0.2} = 36.127s$ $average\ time = \frac{31.6 + 36.127}{2}$ $= 33.9s$	**M1 ecf** **M1 concept of average** **A1**
		Minus 1 for no/missing reasons in Q8	
8	**(a)i**	$\angle FDB$ $= 180° - 77° - 62° - 22°$ ($\angle$s in opposite segments) $= 19°$	**B1**
	aii	$\angle GOB$ $= 360° - [(77° + 62°) \times 2]$ ($\angle$ at centre = $2\angle$ at circumference) $= 82°$	**M1 or 2**$\angle GDB = 2(22° + 19°)$ **A1**
	aiii	$\angle BHD$ $= 62°$ ($\angle$s in the same segment) $\angle HBD$ $= 180° - 90° - 62°$ $= 28°$	**M1** **A1**
	aiv	$\angle DBC$ $= 90° - 28°$ (radius $\perp$ tangent) $= 62°$ $\angle BCD$ $= 180° - 62° - 62°$ $= 56°$	**M1** **A1**

9			
	ai)	$S = \begin{pmatrix} 14 & 9 & 11 \\ 11 & 14 & 10 \\ 8 & 10 & 10 \end{pmatrix}$	B1
	aii)	$P = \begin{pmatrix} 1.5 \\ 1 \\ 2 \end{pmatrix}$	B1
	aiii)	$T = \begin{pmatrix} 14 & 9 & 11 \\ 11 & 14 & 10 \\ 8 & 10 & 10 \end{pmatrix}\begin{pmatrix} 1.5 \\ 1 \\ 2 \end{pmatrix}$ $= \begin{pmatrix} 52 \\ 50.5 \\ 42 \end{pmatrix}$ Amount collect on Monday, Wednesday and Friday respectively	A1 B1
	aiv)	$A = \begin{pmatrix} 0.7 \\ 0.5 \\ 1.1 \end{pmatrix}$	Ba
	av)	$\begin{pmatrix} 14 & 9 & 11 \\ 11 & 14 & 10 \\ 8 & 10 & 10 \end{pmatrix}\begin{pmatrix} 0.7 \\ 0.5 \\ 1.1 \end{pmatrix} = \begin{pmatrix} 26.4 \\ 25.7 \\ 21.6 \end{pmatrix}$ $(1 \quad 1 \quad 1)\begin{pmatrix} 26.4 \\ 25.7 \\ 21.6 \end{pmatrix} = (73.7)$ Profit \$73.70	M1 A1 matrix seen
	b	$-5 - 6y = x$ $4 = -3x - 8$ $x = -4; y = -\frac{1}{6}$	M1 either 1 equation Or $\begin{pmatrix} -5 - 6y \\ 4 \end{pmatrix} = \begin{pmatrix} x \\ -3x - 8 \end{pmatrix}$ A1

10	**(a)**	**(i)**	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= \begin{pmatrix} 3 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} 4 \\ -3 \end{pmatrix}$	**[B1]**
		(ii)	$\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{4^2 + (-3)^2}$ $= 5$ units	**M1** **A1**
		(iii)	Given $\overrightarrow{AC} = 3\overrightarrow{AB}$ $\overrightarrow{OC} - \overrightarrow{OA} = 3\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 12 \\ -9 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ $= \begin{pmatrix} 11 \\ -5 \end{pmatrix}$ Coordinates of *C* are $(11, -5)$.	M1 A1
	(b)	**(i)**	**(a)** $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ **= 4q – p**	A1
			(b) $\overrightarrow{XY} = \frac{1}{2}\overrightarrow{OQ}$ $\quad \overrightarrow{XR} + \overrightarrow{RY} = 2\boldsymbol{q} - 0.5\boldsymbol{p} + 0.5\boldsymbol{p}$ $= \frac{1}{2}(4\mathbf{q})$ $\quad = 2\boldsymbol{q}$ **= 2q**	**A1**
			(c) $\overrightarrow{TQ} = \frac{1}{4}\overrightarrow{OQ}$ $= \frac{1}{4}(4\mathbf{q})$ **= q**	**A1**
			(d) $\overrightarrow{XT} = \overrightarrow{OT} - \overrightarrow{OX}$ $= \frac{3}{4}(4\mathbf{q}) - \frac{1}{2}(4\mathbf{q} - \mathbf{p})$ **= q** + 0.5**p**	**A1**
		(ii)	*XYQT* is a <u>trapezium</u>. Reason : One pair of parallel sides $\overrightarrow{XY} = 2\overrightarrow{TQ}$	**B1** **A1**

11	a	Total $ collected for entry fee $= 5[150(0.8)(1.5) + 150(0.2)(0.8)] +$ $2[300(0.8)(2) + 300(0.2)(1)]$ $= 1020 + 1080$ $=2100$	M1 calculated 20% child and 80% adult M1
		Lifeguard$= 2 \times 5 \times 7 + 13 \times 5 \times 2 \times 6 = 850$ *Maintenance*$= 20 \times 6 \times 2 = 240$ *Water bill*$= 700$ *Total cost of upkeeping 1790*	M1 if lifeguard and maintenance correctly calculated M1 total
		Difference = $2100 - 1790 = 310$ Will make a profit	M1 A1
	b	$\frac{12 \times 0.9 \times 10 + \frac{1}{2}(1.8 + 0.9)(16)(18)}{0.3}$ $= \frac{496.8}{0.3}$ $1656s$ $27 \text{ min } 36s$	M1 for vol M1 for seconds A1
	c		M1 correct shape from 0 m to 0.9 m B1 correct shape from 0.9 to 1.8 m